Putting Matching and Auction Theory to work  
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Description:
This course covers the matching and auction theories that are viewed as a field applying game theory in order to design institutions for the allocation of scarce resources. Matching theory refers to environments where monetary transfers are not available contrary to auctions where the agent are competing through the transfers or prices they are prepared to pay to the designer. Though auctions have a long history, it is only recently that academics are playing the role of consultant to solve complex assignment issues as (1) the use of the spectrum (the FCC spectrum auctions in US and the UMTS auctions for licences for the mobile phone industry), (2) scarce capacities of transmission (e.g. for electricity or gas), (3) the advertising slots of the internet (Google and Yahoo!'s keyword auctions), (4) needs for liquidity (Treasury bounds). Recently economists have also been involved in the design of institutions where prices are absent such as the one governing (1) student placement in schools, (2) labor markets where workers and firms are matched, (3) the organization of organ donation networks, or (4) the housing market. In the US, economists have helped (1) NYC and Boston to design their school choice programs, (2) medical communities to reorganize their hiring procedure, (3) hospitals to organize systematic kidney exchange mechanisms to give kidneys to as many patients as possible and (4) universities to improve house allocation on campus; the list is not exhaustive. Motivated by these applications, the course presents seminal papers and recent development in matching and auction theories with particular emphasis on student placements at school and spectrum auctions.

We start the course with the basics of matching theory. The seminal paper by Gale and Shapley is taught. We present the standard setting of one-to-one matching (matching of couples) and many-to-one matching (matching of students to colleges). The different notions: stability / neutrality / efficiency / ... are introduced and we present the main result of the existence of a stable matching by Gale and Shapley (GS, hereafter). So the underlying mechanism, namely the GS mechanism is presented and studied. In particular, it is shown that the GS mechanism is weakly preferred by every student to any stable matching; and that "The set of students and colleges that are unmatched is the same for all stable matchings." The incitation for individuals to tell the truth to the clearinghouse is then studied. The notion of truth-telling in dominant-strategy, or equivalently strategyproofness, is studied and this part ends by a discussion of the impossibility result by Roth (1982): "There is no stable mechanism that is strategy-proof."

Using the above theory we present the recent developments on school choice i.e. the placement of students in schools. Abdulkadiroglu and Sonmez (2003) showed that placement mechanisms used in many cities such as Boston are flawed, and proposed new mechanisms to improve upon existing placement mechanisms. We present why -- based on this and other studies -- Boston and New York City changed their student placement mechanisms. Based on Gale and Shapley's setting, we present the standard model of the school choice problem. The problematic features of the Boston mechanism are analyzed. In particular, Chen and Sonmez (2005)'s experimental evidence on preference manipulation under the Boston mechanism is
discussed. The modified mechanism proposed (based on GS's mechanism) by Abdulkadiroglu and Sonmez (2003) is presented. Strategy proofness (and group strategyproofness) on the side of the students is proved and it is proved that it is actually the unique mechanism that is both strategyproof and stable. The results of the changes from Boston mechanism to Abdulkadiroglu and Sonmez (2003)'s mechanism are presented. Inefficiency of stable mechanisms (compared to unstable ones) is discussed, and based on this, we discuss an alternative mechanism (based on the Top Trading Cycle algorithm by Gale). This latter mechanism is studied and it is shown in particular that it characterizes strategy-proofness, Pareto-efficiency and individual rationality. We also discuss when exactly the GS mechanism is inefficient. The trade-off between stability (GS) and efficiency (TTC) is presented. Finally, recent developments by Abdulkadiroglu, Che and Yasuda (2008) and Erdil and Ergin (2008) are discussed: when we use matching theory for labor markets, strict preferences seemed like a good assumption, but in school choice, indifferences (weak priorities) are important: A new issue in school choice is how to break ties. Tradeoffs between efficiency, stability, and strategy-proofness are found: GS with any tie breaking is still strategy-proof. The outcome of a GS may not be a student-optimal stable matching: there may be a stable matching that is better for everyone.

Other issues that the course may address are the organization of (i) organ donation networks and (ii) the housing market. In 2004, New England approved the establishment of a clearinghouse for kidney exchange. Game theorists as well as doctors designed the clearinghouse. We survey the issues of the kidney exchange problem and also detail the mechanisms now at use in New England. Results on how to maximize the number of transplantations are reported. For the housing market, the setting by Hylland and Zeckhauser (1979) is introduced. The serial dictatorship mechanism is analyzed. In particular, for the standard on-campus housing problem, the critiques of the serial dictatorship mechanism based on Abdulkadiroglu and Sonmez (1999) are discussed. Alternative solutions are presented.

We follow the course with the basics of mechanism design. The different notions used in the literature are presented. We provide conditions under which the revelation principle holds i.e. where we can restrict our attention to simple (direct) mechanisms. The impossibility theorem by Gibbard-Satterthwaite is presented i.e. if we do not restrict the set of possible preferences for agents, under fairly weak assumptions, a social goal is strategy-proof if and only if it is dictatorial. By restricting the set of possible preferences, we present the Vickrey-Clark-Groves (VCG) mechanism that "goes around" Gibbard-Satterthwaite's impossibility theorem. A general characterization of the social goals that are strategy-proof is introduced. The payoff equivalence principle is presented. The revenue equivalence theorem is then obtained from payoff equivalence for the standard symmetric auction setup. We then deduce an influential impossibility result: Myerson and Satterthwaite (83): there is no efficient way for two parties to trade a good when they each have secret and probabilistically varying valuations for it, without the risk of forcing one party to trade at a loss. The optimal auction is derived for the asymmetric independent private value (IPV) model: Myerson (81)'s techniques are carefully taught. The VCG mechanism is illustrated in multi-unit environments of an homogenous good and with multiunit demand: Ausubel's ascending auction (which is patented) illustrates the construction. We then present the impossibility result by Jehiel and Moldovanu (2001): efficient Bayesian mechanisms do not exist generically in environments with multidimensional signals and interdependent values.
We then move to standard single-unit formats that are commonly used and show how to derive equilibrium bidding behaviors with symmetric bidders. The role of reserve prices and entry fees are discussed. The distortion implied by asymmetric bidders is discussed and also the links with the mechanism design theory. A critical look is then adopted by revisiting the seminal contributions when entry is endogenous.

Then, we consider environments with interdependent values. First the general symmetric interdependent value model of Milgrom and Weber (82) and the related analysis of standard auction formats are presented. The revenue ranking between standard auction formats is presented and also extended to environments with endogenous entry. Second, two extensions are developed: one where an informed bidder competes with uninformed bidders, a popular model for drainage auctions, and one where the seller participates (possibly anonymously) in the auction, a pervasive phenomenon that occurs not only in online auctions but also in art or real estate auctions.

Environments with allocative externalities seem to be of first order for the sale of licenses (e.g. spectrum) or take-overs. Two new insights may occur in such environments: endogenous valuations and strategic nonparticipation which modify the analysis of standard auction design where efficiency is no more guaranteed under complete information. The auction that maximizes the revenue by means of optimal threats is presented. We apply the insights from this strand of the literature by considering an economic policy case study: the choice of the number of licenses to auction for UMTS licenses. The performance and the details of the auction design for several European countries are discussed.

Finally, we turn to multi-objects auctions, mainly in private values environments. Standard issues as demand reduction or the exposure problem are presented with simple examples. We move to alternative formats that can alleviate those issues: combinatorial auction format as the first package price (also called menu auction) or Ausubel & Milgrom proxy auction that implements the Vickrey outcome under simple restrictions, finally Google's ad auction (also called position auctions).

Some general non-technical bibliography:

Classic textbooks:

The basics of matching theory

**School choice**


**Housing market**


**Kidney exchange**


**Basics of mechanism design**


**Bidding equilibrium analysis in standard auctions**


**Auctions with informational externalities**


Laurent Lamy (2009), "The Shill Bidding Effect versus the Linkage Principle", Journal of Economic Theory 144, 390-413

**Auctions with allocative externalities & Multi-objects auctions**


